Leaving an Interval in Limited Playing Time

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A player starts at x in $(-G, G)$ and attempts to leave the interval in a limited playing time. In the discrete time problem, G is a positive integer and the position is described by a random walk starting at integer x, with mean increments zero, and variance increment chosen by the player from $[0, 1]$ at each integer playing time. In the continuous time problem, the player's position is described by an Ito diffusion process with infinitesimal mean parameter zero and infinitesimal diffusion parameter chosen by the player from $[0, 1]$ at each time instant of play. To maximize the probability of leaving the interval $(-G, G)$ in a limited playing time, the player should play boldly by always choosing largest possible variance increment in the discrete-time setting and largest possible diffusion parameter in the continuous-time setting, until the player leaves the interval. In the discrete time setting, this result affirms a conjecture of Spencer. In the continuous time setting, the value function of play is also identified.

Some Stochastic Control Problems and their Applications to Inequalities for Diffusions

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The problem of controlling a Brownian motion with a process of bounded velocity, so as to minimize certain functionals of its path, is considered. In particular the functionals

$$E[f(x_t)],\quad E\left[\int_0^\infty e^{-\alpha s} f(x_s)\,ds\right]\quad\text{and}\quad E\left[f\int_0^T I[x_s>t]\,ds\right],$$

for symmetric, increasing for $x \geqslant 0$ functions f, are minimized. Stochastic bounds for a class of diffusions are thus founded, and this result is generalized to n-dimensional processes. Similarly, stochastic bounds for T_t, the time $|x_t|$ spends above a certain level are found for the same class of diffusions. Explicit solutions are given for the first two problems.

Optimal Switching between a Pair of Brownian Motions

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Consider a pair of Brownian motions X_s and Y_t on the interval $[0, 1]$ with absorption at the endpoints. The joint state space is the square $E = [0, 1] \times [0, 1]$. The time evolution of the two processes can be controlled separately: i.e., we can let the X_s process run and freeze the Y_t process to obtain horizontal Brownian motion, or we can let the Y_t process run and freeze X_s giving us vertical Brownian

motion. We assume that there is a pay-off function $f(x, y)$ that is zero in the interior of E and non-negative on the boundary of E. The objective is to find the optimal strategy for controlling the time evolution and a corresponding stopping time so as to maximize the expected pay-off obtained at the time of stopping. The optimal strategy is determined by a partition of the state space into three sets: horizontal control, vertical control, and stop. We will give a rather explicit characterization of these sets.

Stochastic Sequential Assignment based on Discrete Match-Levels

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M "offers" (e.g. kidneys for transplants) arrive in a random stream and are to be sequentially assigned to N waiting candidates. Each candidate, as well as each arrival, is characterized by a random attribute drawn from a discrete-valued probability distribution function. An assignment of an offer to a candidate yields a reward $r(0)$ if they match, and a reward $r(1) \leq r(0)$ if not. We derive optimal sequential assignment policies which maximize the expected total reward for various cases where $M \neq N$ and various decay assumptions on the underlying life time distribution of the process. We give intuitive explanations for these optimal strategies and indicate applications.

2.5. Estimation and statistical inference

The Role of Martingales in the Analysis of Biomedical Data

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The theory of martingales has become a useful tool in the construction of estimating equations, and associated inference problems. It is particularly useful for non-parametric inference, but is increasingly found to be useful in parametric inference for processes which are only partially observed. The present paper reviews applications in the analysis of infectious disease data, survival/sacrifice data and capture-recapture experiments.

Optimality of Sequential Probability Ratio Tests for a Class of Continuous Parameter Stochastic Processes

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For a wide class of stochastic processes (s.p.) including processes belonging to exponential families, it is proved that Wald SPRT is optimal in the sense of minimizing the expectation of an increasing process associated with the s.p.